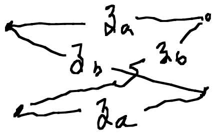
schedule presentations; final exam
poles of zij
lattice vs bridge tee & Constant R examp
OTA circuit

exp(-At) and delta_x(0)
sesitivity
adjoint

V2/V1(s) = $[1-(z_a/R)^2]/[1+2(z_a/R)+(z_a/R)^2] = \{[1-(z_a/R)][1+(z_a/R)]/[1+(z_a/R)]^2 = [1-(z_a/R)]/[1+(z_a/R)]$

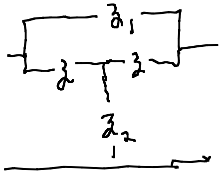
$R^2 = z_a z_b$ <=> $G^2 = y_a y_b$ G=1/R



$$Z = \frac{1}{2}\begin{bmatrix} z_b+z_a & z_b-z_a \\ z_b-z_a & z_b+z_a \end{bmatrix} \quad det = \frac{1}{4} z_a z_b$$

$$Y = \frac{1}{det Z} \cdot \frac{1}{2}\begin{bmatrix} z_b+z_a & z_a-z_b \\ z_a-z_b & z_a+z_b \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} y_b+y_a & y_b-y_a \\ y_b-y_a & y_b+y_a \end{bmatrix}$$



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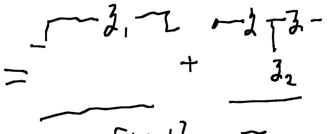

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$Y_1 = y_1\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ $\quad Z_2$

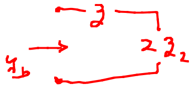
$$Z_2 = \begin{bmatrix} z+z_2 & z_2 \\ z_2 & z+z_2 \end{bmatrix}; \quad det Z_2 = z^2 + 2 z_2 z$$
$$= z(z+2z_2)$$

$$Y = y_1\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + \frac{1}{z(z+2z_2)}\begin{bmatrix} z+z_2 & -z_2 \\ -z_2 & z+z_2 \end{bmatrix} \Rightarrow y_b = \cancel{y_1} + \frac{z+z_2}{z(z+2z_2)} \cdot \frac{-z_2}{z(z+2z_2}$$ $$= \frac{z}{z(z+2z_2)}$$

bridge tee.

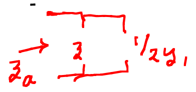
$$= \frac{1}{z+2z_2} = \frac{1}{z_b}$$

$\Rightarrow$



$$\left.\begin{aligned} y_{11} &= \tfrac{1}{2}(y_b+y_a) \\ y_{21} &= \tfrac{1}{2}(y_b-y_a) \end{aligned}\right\} \quad \begin{aligned} y_b &= y_{11}+y_{21} \\ y_a &= y_{11}-y_{21} \end{aligned}$$

$y_a = 2y_1 + \frac{z+2z_2}{z(z+2z_2)} = 2y_1 + 1/z \Rightarrow$



∴ given bridge tee can get a lattice

if $z_1 = 1/y_1$, z_2, z are PR both circuits are passive

For constant R, $\det_z = R^2 = z_a z_b$, $G = 1/R$

$\det Y = \frac{1}{\det Z} = \frac{1}{\det Y} = \frac{1}{G^2} = \frac{1}{y_a} \cdot \frac{1}{y_b}$

$\Rightarrow G^2 = y_a y_b$

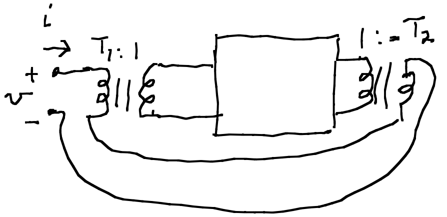
$G^2 = \frac{y_1 z(z+z_2) + z_2}{z(z+2z_2)} \frac{y_1(z+2z_2)+1}{z(z+2z_2)}$

if $y_a y_b = G^2 \Rightarrow G^2 z^2 + 2G^2 z z_2 = 1 + 2y_1 z \Rightarrow z^2 + (2z_2 - 2y_1/G^2) - 1/G^2 = 0 \Rightarrow$ gives z

$z = (z_2 - y_1/G^2) \pm \frac{1}{2}\sqrt{4(z_2 - y_1/G^2)^2 + 4G^2}$ if $y_1/G^2 = z_2 = \frac{1}{y_2} \Rightarrow G^2 = y_1 y_2$

then $z = 0 \pm G \Rightarrow z = G$ if passive, $G > 0$

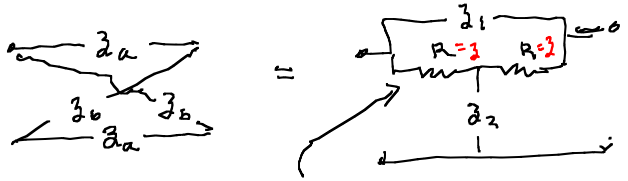
To show poles of z_{12} & z_{21} in z_{11} & z_{22} if passive



$\frac{v}{i} = [T_1 \;\; T_2]\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}\begin{bmatrix} T_1 \\ T_2 \end{bmatrix}$

$= T_1^2 z_{11} + (z_{12} + z_{21}) T_1 T_2 + z_{22} T_2^2$

$\therefore$



if const R, $z=R$

$y_a = 2y_1 + 1/z$

$z_b = 2z_2 + z$

- - -

$\dot{x} = Ax + Bu$
$y = Cx + Du$

if $\underset{\text{input}}{\underset{\shortparallel}{u = v}}$, $\underset{\text{output}}{\underset{\shortparallel}{y = i}}$ then the transfer function is an admittance

x = capacitor voltage ; $\frac{dx}{dt} = \dot{x} \Rightarrow$ capacitor current

$C \cdot 1_k$, k = dimension of state

$$\underset{\uparrow \atop \text{currents}}{\begin{bmatrix} \dot{x} \\ y \end{bmatrix}} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \underset{\uparrow \atop \text{voltages}}{\begin{bmatrix} x \\ u \end{bmatrix}} \Rightarrow Y_c = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

$\leftarrow$ admittance entries make with OTAs

state variable equations can be made with OTAs for Y_C & capacitors for $\dot{x}$

if $u=0$ then $\dot{x} = Ax$ if $x = e^{-At}x_0$ then $\dot{x} = \frac{d(e^{-At})}{dt} \cdot x_0$ need to know e^{-At}

for a scalar $e^{w} = 1 + \frac{1}{1}w + \frac{1}{2}w^2 + \cdots = \sum_{i=0}^{\infty} \frac{1}{i!} w^i \Rightarrow e^{-At} = \sum_{i=0}^{\infty} \frac{1}{i!}(-At)^i = \sum_{i=0}^{\infty} \frac{(-1)^i}{i!} A^i t^i$; $\frac{de^{-At}}{dt} = \sum \frac{(-1)^i A^i i t^{i-1}}{i!}$

$= -A \sum_{i=1}^{\infty} \frac{(-1)^{i-1}}{(i-1)!} t^{i-1} = -A e^{-At}$

$\dot{x} = Ax + \delta x_0$ $\quad$ $\delta(t) = \frac{d\,1(t)}{dt}$: unit impulse $\quad$ k = finite means $(s1_k - A)^{-1}$ has k poles

$\Rightarrow$ a finite number of poles

$s1_k\,\mathcal{L}[x] = A\,\mathcal{L}[x] + x_0 \Rightarrow \mathcal{L}[x] = (s1_k - A)^{-1}x_0$

to get $x(t)$ we invert using the poles

Ex: $\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ -2 & -3\end{bmatrix} x + \delta\begin{bmatrix}x_{0_1}\\ x_{0_2}\end{bmatrix}$; $\mathcal{L}[x] = \left(\begin{bmatrix}s & 0\\ 0 & s\end{bmatrix} - \begin{bmatrix}0 & 1\\ -2 & -3\end{bmatrix}\right)^{-1} x_0$

$$\begin{bmatrix}s & -1\\ 2 & s+3\end{bmatrix}^{-1} = \frac{1}{s(s+3)+2}\begin{bmatrix}s+3 & +1\\ -2 & s\end{bmatrix} = \frac{1}{s^2+3s+2}\begin{bmatrix}s+3 & +1\\ -2 & s\end{bmatrix} = \frac{1}{s+2}\begin{bmatrix}\overset{-1}{k_{11}} & \overset{1}{k_{12}}\\ \underset{2}{k_{21}} & \underset{2}{k_{22}}\end{bmatrix} + \frac{1}{s+1}\begin{bmatrix}\hat{k}_{11} & \hat{k}_{12}\\ \hat{k}_{21} & \hat{k}_{22}\end{bmatrix}$$

$s^2 + 3s + 2$

$s = -\frac{3}{2} \pm \frac{1}{2}\sqrt{9-8}$

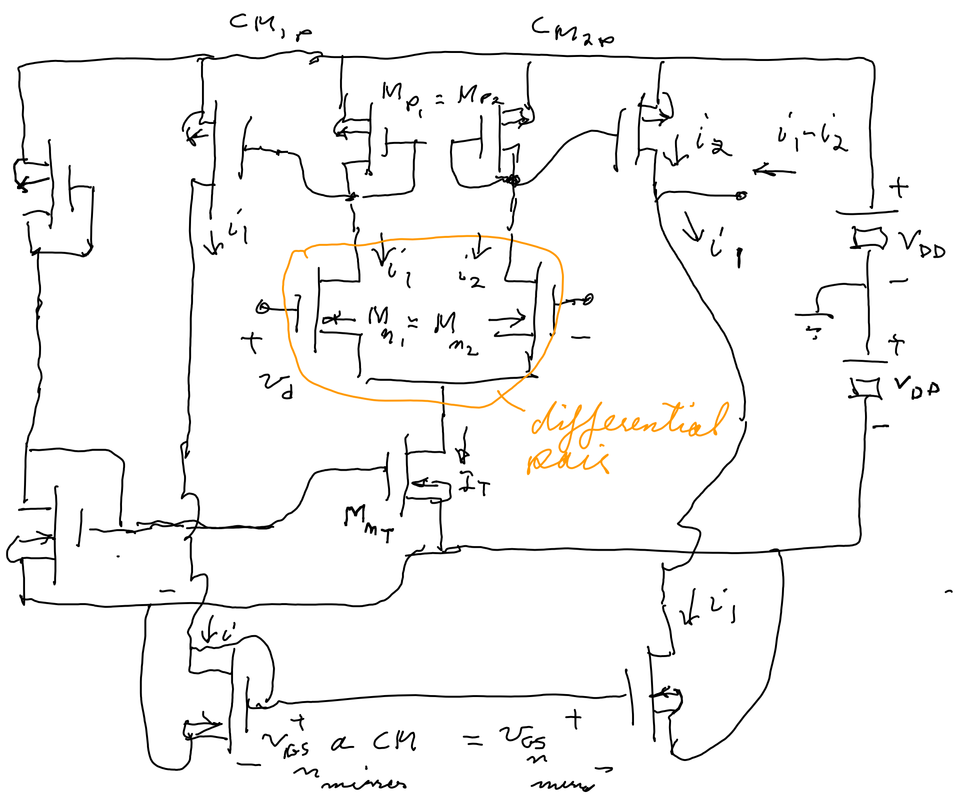
$= -3/2 \pm \frac{1}{2} = -2, -1$

$k_{11} = \frac{s+3}{(s+2)(s+1)} \times (s+2)\Big|_{s=-2} = \frac{1}{-1}$ $\quad$ $k_{21} = \frac{-2}{s+1}\Big|_{s=-2} = 2$ $\quad$ $k_{12} = \frac{-1}{-1} = 1$, $k_{22} = \frac{-2}{-1} = 2$

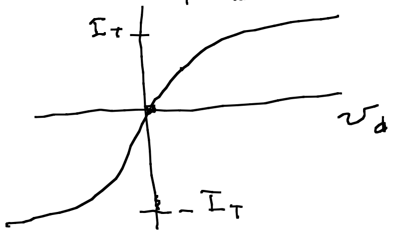
$\hat{k}_{11} = \frac{2}{1} = 2$, $\hat{k}_{21} = \frac{-2}{1} = -2$, $\hat{k}_{12} = \frac{+1}{1} = +1$, $k_{22} = \frac{-1}{1} = -1$

$$\Rightarrow \begin{bmatrix}s & -1\\ 2 & s+3\end{bmatrix}^{-1} = \frac{1}{s+2}\begin{bmatrix}-1 & 1\\ 2 & 2\end{bmatrix} + \frac{1}{s+1}\begin{bmatrix}2 & +1\\ -2 & -1\end{bmatrix} \Rightarrow e^{-At}1(t) = \left\{\begin{bmatrix}-1 & 1\\ 2 & 2\end{bmatrix}e^{-2t} + \begin{bmatrix}2 & +1\\ -2 & -1\end{bmatrix}e^{-t}\right\} 1(t)$$

for $s = \sigma + j\omega$, $\sigma > 0$



an OTA



a good VLSI circuit

$$i_1 - i_2 = I_T \tanh\left(\frac{v_d}{2V_T}\right)$$

V_T = thermal voltage

if M_{n_1} & M_{n_2} ⇒ BJT